Scaling behaviour in daily air humidity fluctuations

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We show that the daily average air humidity fluctuations exhibit non-trivial $1/f^\alpha$ behaviour which different from the spectral properties of other meteorological quantities. This feature and the fractal spatial structure found in clouds make it plausible to regard air humidity fluctuations as a manifestation of self-organized criticality. We give arguments why the dynamics in air humidity can be similar to those in sandpile models of SOC.

There is widespread interest in understanding the dynamic processes of large, spatially extended systems. There are many geophysical processes in nature where scale-invariance is observed. Recently, the Gutenberg-Richter law of earthquakes [1], the volcanic activity [2] and various astrophysical phenomena [3] have been reported to show scale-invariant properties. However, such quantitative analysis is often limited by a shortage of available data or a lack of understanding of the basic effects. One of the largest extended systems is our atmosphere. Although all the basic mechanisms that govern the dynamics of the atmosphere have been well known for quite a long time, a detailed understanding and adequate characterization of the fluctuations of various statistical quantities of the lower atmospheric boundary layer are still not complete. The conventional viewpoint is that the underlying mechanism is random and can be considered as an autoregressive process. Quite surprisingly, in Ref. [4] it was possible to demonstrate that the distribution and spectral properties of the daily medium temperature fluctuations coincide with those measured in a helium cell in Refs. [5,6]. The daily mean temperature fluctuations on a one day scale and the *Soft Turbulent* 2K helium cell fluctuations on a millisecond scale look the same since the dimensionless parameters, the Prandtl and Raylegh numbers of the two systems, are the same. The main objection to using daily average data for deeper analysis is the belief that random weather changes (fronts, etc.) dominate the temporal behaviour of the quantities measured at a typical meteorological station. In reality, in the case of daily temperature fluctuations, it was observed that the meteorological fronts due to the large scale coherent spatial motion affect only the short (1-10 days) timescale, while the vast majority of timescales (10-10000 days) is governed by the inherent fluctuations of the Benard-Raylegh convection in the lower 500m boundary layer [4]. In this Letter we would like to show that the fluctuations of the daily average relative air humidity data also reflect some inherent physical processes and that this measurement is an ideal candidate for observing large scale-invariance in the atmosphere.

The air humidity can be quantified by the partial vapor pressure p_v of gaseous water in the air. At a given temperature T, the amount of water in the air is limited. The maximum partial vapor pressure corresponding to the maximum water content is the saturation vapor pressure $p_{sat}(T)$. The relative air humidity e at temperature T is the ratio of the actual to the saturation pressure measured in percent

$$e = \frac{p_v}{p_{sat}(T)} \cdot 100\%. \tag{1}$$

A detailed summary of the measurement and the definition of the air humidity and the saturation vapor pressure are given in the classic work of Penman [7].

The special importance of the humidity from the point of view of self-organized criticality (SOC) [8] lies in its threshold behaviour. If the actual humidity in the air locally exceeds 100%, the water starts condensing. In the absence of macroscopic spatial motion of the air, the microscopic amount of condensed water is transported on microscopic scales by diffusion. It can reach a new location, where the local relative humidity is under 100% and can return to the gaseous state. If the humidity in the neighbourhood is also saturated, the amount of condensed water will increase. This can initiate a chain reaction like the avalanches in the 'sandpile' models [8]. We propose that large decreases in the air humidity are consequences of such processes. On the other hand, the humidity in the air is coming from open water surfaces, from the moisture in soil and from plants and animals. If the air humidity is under 100%, the evaporation from these sources continually increases the water content of the air. This is the analog of dropping sand on the sandpile, and this drives the system toward the fully saturated

100% humidity state which is the stationary and critical state of the system at the same time. It is very likely that these aspects of the dynamics help to build up a self-organized critical state on large scales. The two main characteristics of the SOC behaviour is the $1/f^\alpha$ type behaviour of the power spectra of the fluctuations, and the spatial scaling behaviour manifested in the formation of fractal structures [8]. One of the most well known fractals in nature are clouds [9], which are formed in the chain reaction-like condensation of air humidity. These processes and large scale fractal behaviour of clouds can be observed in meteorological satellite pictures [10]. The condensation of air humidity into clouds can be modeled by cellular automata, which has recently been reported to show SOC behaviour [11].

To detect scaling in the temporal behaviour, we have analyzed relative air humidity data collected by the Hungarian Meteorological Service in various stations in Hungary during the period 1963 - 1988. The main difficulty in analyzing meteorological time series is that seasonal periodic variations can cause peaks in the Fourier transforms and make it difficult to determine the power spectra of the fluctuations. Figure 1 shows the seasonal variation $\bar{e}(i)$ of a typical station (Balaton, county Heves), which is defined as

$$\bar{e}(i) = \frac{1}{26} \sum_{y=1963}^{1988} e(i,y), \quad i = 1, ..., 365 \tag{2}$$

where $e(i,y)$ denotes the daily average air humidity of the ith day of the yth year measured at the meteorological station. To diminish the effect of seasonal variations we subtract it from the daily data. Then the fluctuation e_f is given by the deviation of the actual humidity from the seasonal variation (2):

$$e_f(i,y) = e(i,y) - \bar{e}(i), \tag{3}$$

where the notation is the same as above. In this way we obtain a 26×365 day long record of daily average relative air humidity fluctuations. Hungary can be considered as meteorologically homogeneous from the point of view of fluctuations [12] due to the geography. Different stations measure coherent fluctuations [4], and one station can be considered as representative. Figure 2 shows a typical record of the daily average humidity fluctuations for the year 1969. Figure 3 shows a histogram of the daily average humidity fluctuations, which is clearly a Gaussian distribution, with zero mean and standard deviation of $\sigma = 9.34\%$. The power spectrum $S(f)$,which is the squared magnitude of the Fourier amplitude, can be obtained by standard numerical methods [13]. Figure 4 shows the power spectrum of the time series measured in Balaton. Almost the whole frequency range can be fitted by a simple power law

$$S(f) \sim \frac{1}{f^\alpha}, \tag{4}$$

with $\alpha = 0.61 \pm 0.01$. In addition to the large scale spatial scaling behaviour this shows the existence of scaling behaviour also in the time domain. A very nice feature of this result is that meteorological fronts and other uncorrelated events do not seriously affect the spectra. If the underlying process is SOC, the exponent α can be related to the scaling of the weighted average of the avalanches [14]. The weighted average is defined as

$$\Lambda(t) = \sum_s s^2 P(s,t), \tag{5}$$

where $P(s,t)$ is the distribution of the avalanche sizes s as a function of the duration t of the avalanches. For SOC this scales as

$$\Lambda(t) \sim t^\mu, \tag{6}$$

where the exponent satisfies [14] $-1 < \mu < +1$. The exponents μ and α are related [14] as $\alpha = \mu + 1$, which yields $\mu = -0.39 \pm 0.01$.

Finally, to demonstrate the non-trivial nature of a $1/f^\alpha$ type power-density spectrum, on Figures 5 and 6 we show the power spectra of the daily temperature and rain fluctuations respectively, measured at the same meteorological station. The temperature record has been analyzed in Ref. [4], while the rain record appears to be white noise (ie. a constant power spectrum). Neither show any sign of scaling behaviour.

In summary, we have found that the spectral density of daily average air humidity fluctuations shows scaling behaviour. This is very different from the spectral properties of other meteorological quantities. This and the fractal behaviour of the condensed air humidity in space make it very plausible to regard air humidity fluctuations as a manifestation of self-organized criticality. We gave arguments why the dynamics in air humidity can be similar to those in sandpile models of SOC. We hope that this study can inspire more detailed analysis of the space-time dynamics of clouds and air humidity fluctuations.

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FIG. 1. The seasonal variation of the daily average air humidity (Eq.(2)) obtained from the data of the meteorological station Balaton, county Heves, Hungary for the period 1963-1988.

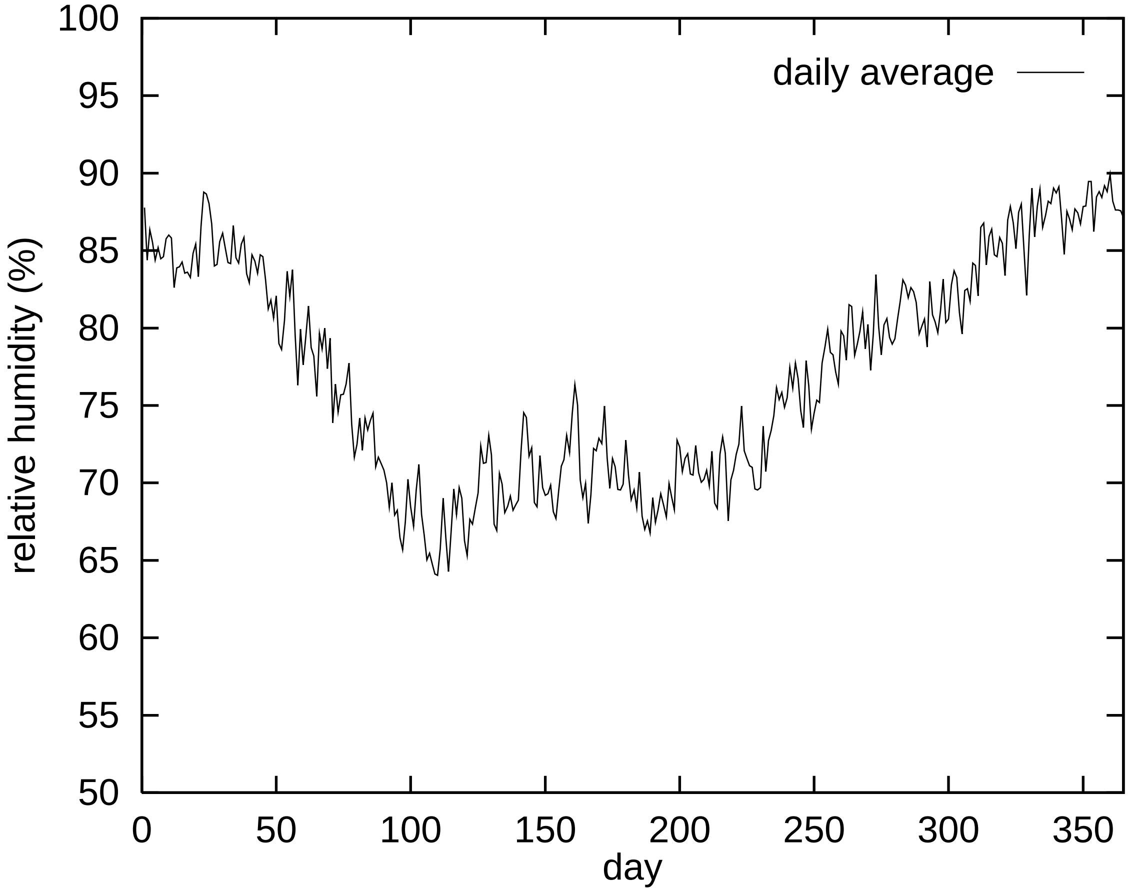
FIG. 2. Air humidity fluctuations (Eq.(3)) measured in 1969.

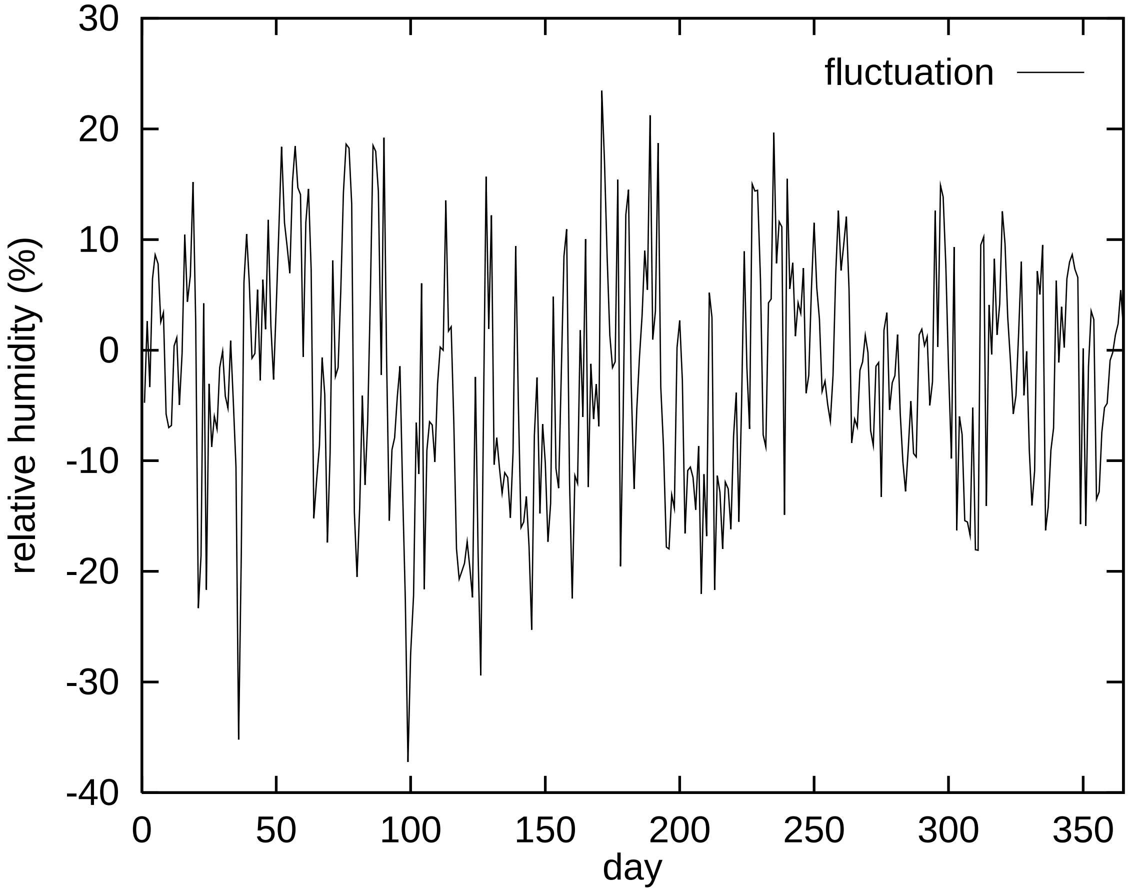
FIG. 3. Unnormalized histogram of the air humidity fluctuations measured in the period 1963-1988. The dashed line is the fitted Gaussian distribution with standard deviation $\sigma = 9.34\%$

FIG. 4. Unnormalized power spectrum of the daily average relative air humidity fluctuations. The dashed line is the best fit given by (Eq.(4)).

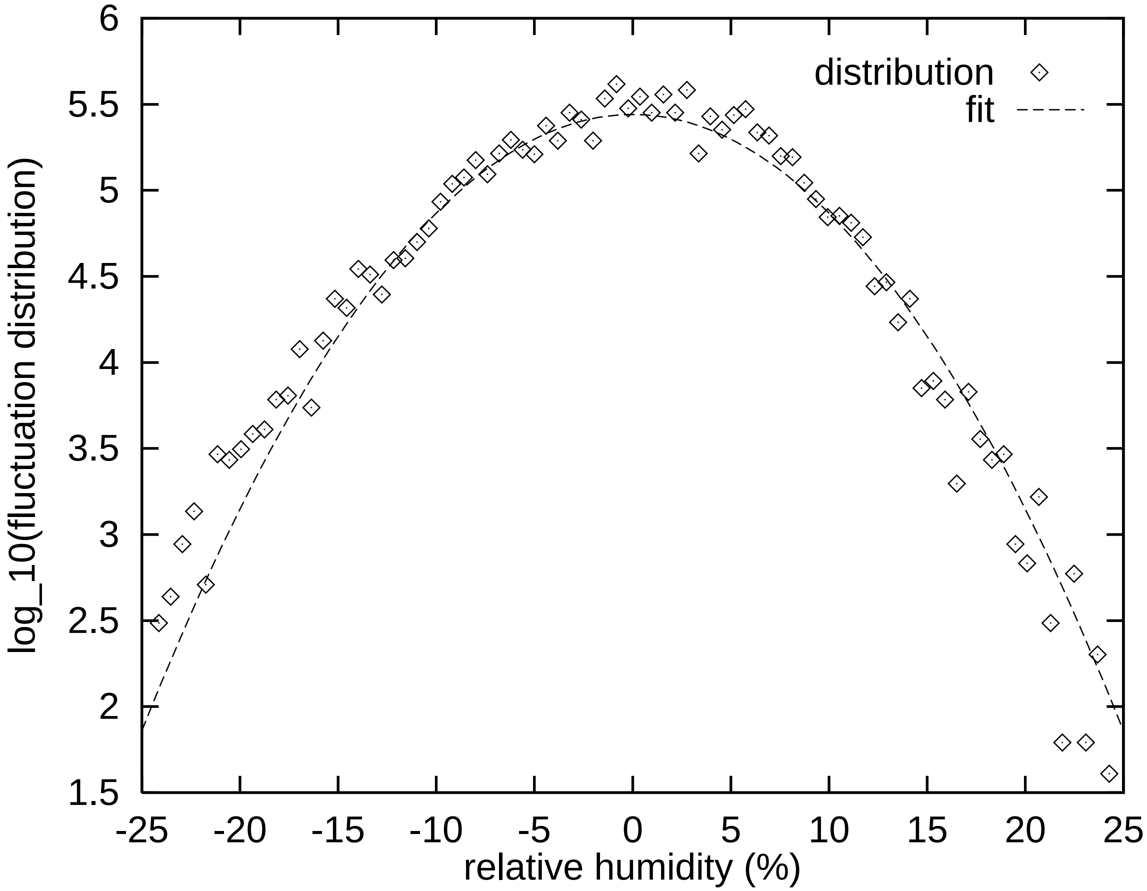
FIG. 5. Unnormalized power spectrum of the daily medium temperature fluctuations.

FIG. 6. Unnormalized power-density spectrum of the daily rain fluctuations.

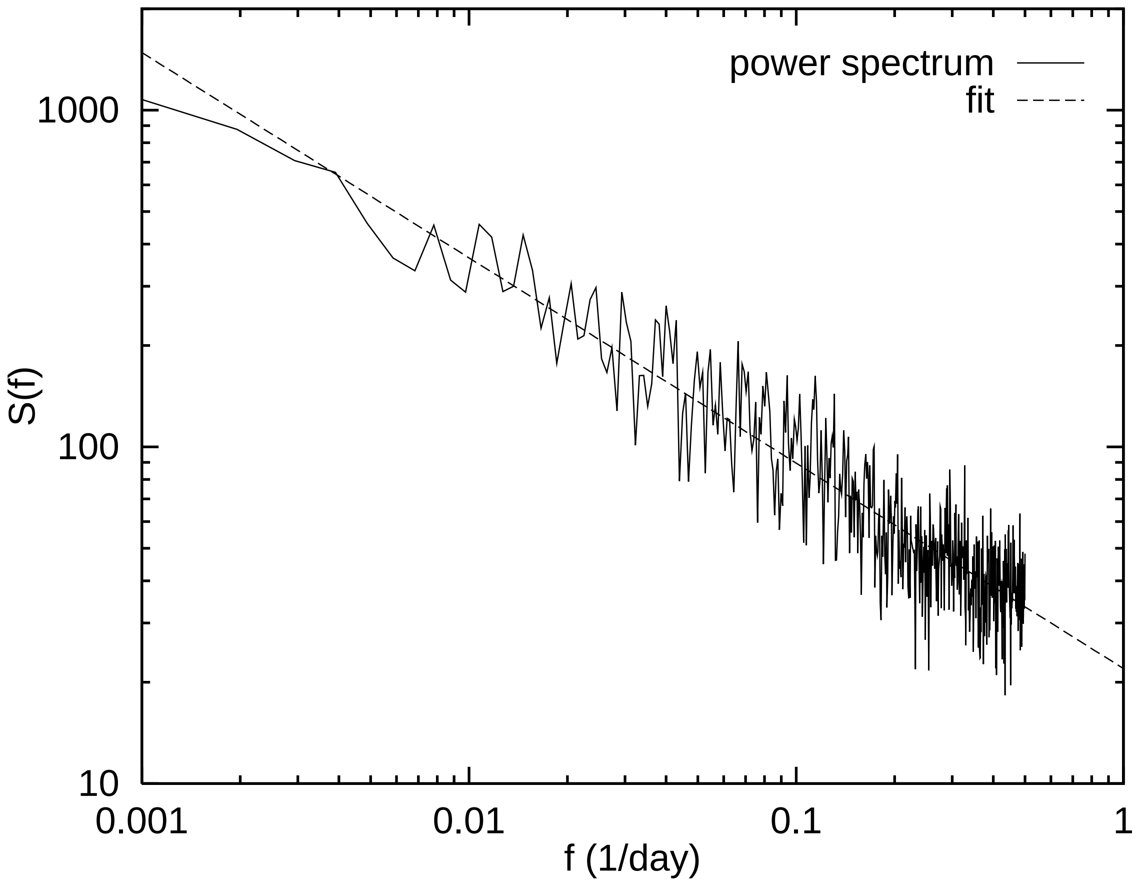

daily average
relative humidity (%)
100
95
90
85
80
75
70
65
60
55
50
0
50
100
150
200
250
300
350
day



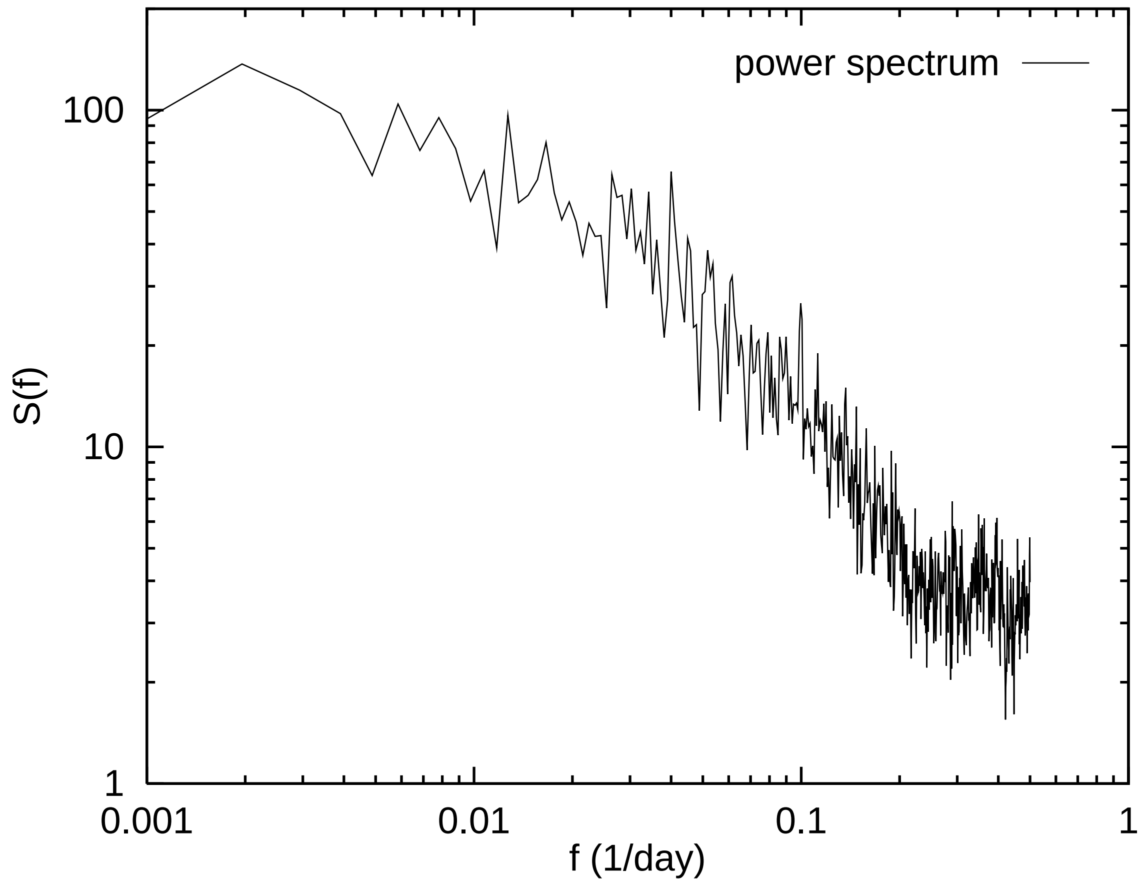
fluctuation
relative humidity (%)
day
30
20
10
0
-10
-20
-30
-40
0
50
100
150
200
250
300
350

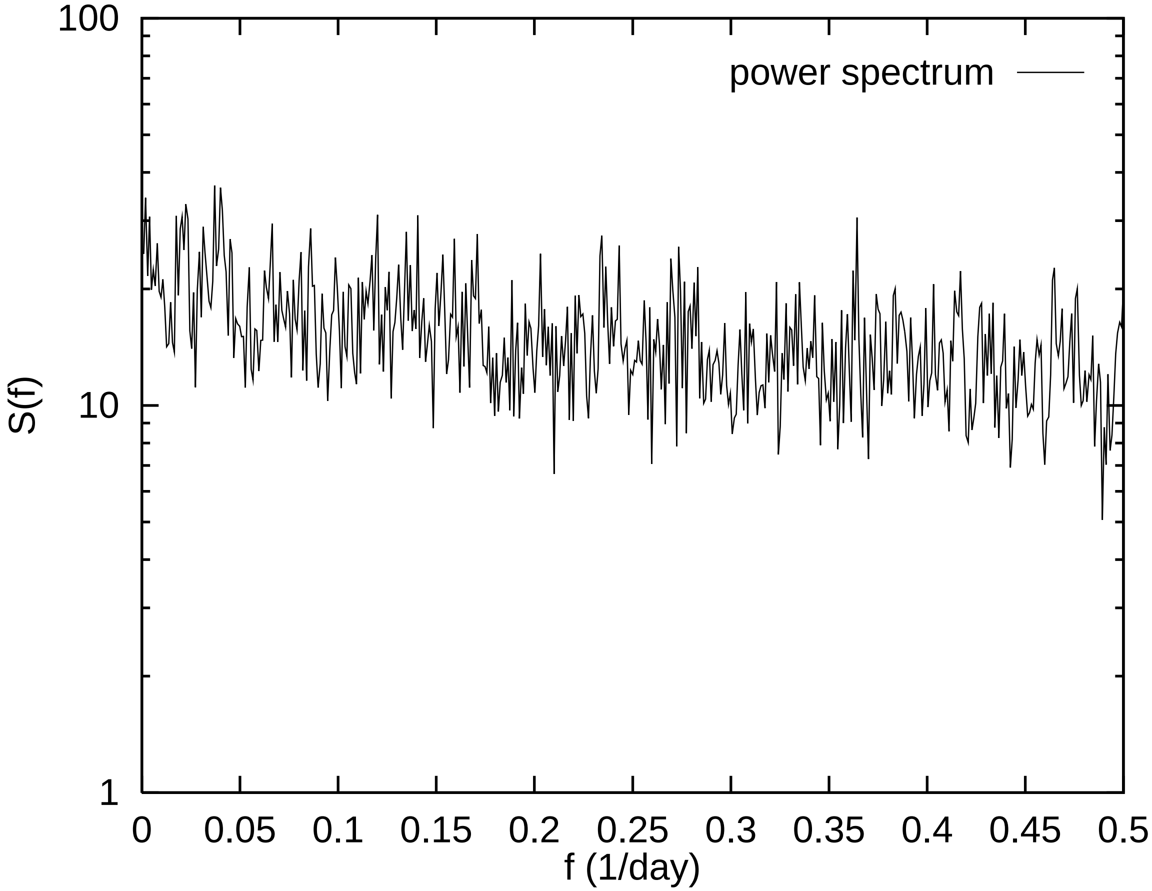


distribution
fit
log_10(fluctuation distribution)
relative humidity (%)
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
-25
-20
-15
-10
-5
0
5
10
15
20
25



power spectrum
fit
S(f)
1000
100
10
0.001
0.01
0.1
1
f (1/day)


power spectrum
100
10
1
S(f)
0.001
0.01
0.1
1
f (1/day)


power spectrum
S(f)
f (1/day)
100
10
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5